Mechanics II

FIZIKA SJPO Training

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Contents

1 Notes

1.1 Work, Energy & Power

So far, we have only dealt with forces. However, we can also turn to energy as another important quantity in dynamics. And, associated with energy, we have work and power.

1.1.1 Work Done By A Force

The **work done** W by a constant force $\boldsymbol{F}$ through a displacement $\boldsymbol{s}$ is defined as:

$$W = \boldsymbol{F} \cdot \boldsymbol{s} = |\boldsymbol{F}|\,|\boldsymbol{s}|\cos\theta \tag{1}$$

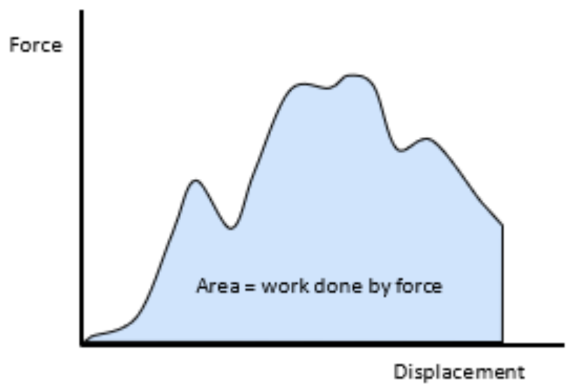
where θ is the angle between $\boldsymbol{F}$ and $\boldsymbol{s}$.

You will most commonly encounter the cases of $\theta = 0°, 90°$ and $180°$.

If the force $\boldsymbol{F}$ is not constant, but is rather constant over intervals, we can write:

$$W = \sum_i \boldsymbol{F}_i \cdot s_i = \sum_i |\boldsymbol{F}_i|\,|\boldsymbol{s}_i|\cos\theta_i \tag{2}$$

Graphically, W is the area under the force-displacement graph:



Example 1.1. A force acts for 10 s on an object of 2 kg initially at rest. It is the only force acting on the object and it has a constant direction. The table below shows the magnitudes of the force at the following time intervals. Find the total work done by the force on the object.

Time	Magnitude
0 – 2 s	2 N
2 – 5 s	4 N
5 – 10 s	8 N

Solution. Since we want to find work, we need to somehow deduce the displacements during each time interval first. We can do this by considering the kinematics of the situation:

$$x = v_{\text{i}}t + \frac{1}{2}at^2 = v_{\text{i}}t + \frac{F}{2m}t^2$$

$$v_{\text{f}} = v_{\text{i}} + at = v_{\text{i}} + \frac{F}{m}t$$

Now, we can slowly calculate all the required quantities:

$$x_{0\text{ to }2} = \frac{2}{2(2)}(2)^2 = 2\,\text{m}$$

$$v_2 = \frac{2}{2}(2) = 2\,\text{m/s}$$

$$x_{2\text{ to }5} = 2(3) + \frac{4}{2(2)}(3)^2 = 15\,\text{m}$$

$$v_5 = 2 + \frac{4}{2}(3) = 8\,\text{m/s}$$

$$x_{5\text{ to }10} = 8(5) + \frac{8}{2(2)}(5)^2 = 90\,\text{m}$$

Hence, the total work done can be evaluated as per Equation (2):

$$W = F_{0\text{ to }2}x_{0\text{ to }2} + F_{2\text{ to }5}x_{2\text{ to }5} + F_{5\text{ to }10}x_{5\text{ to }10} = (2)(2) + (4)(15) + (8)(90) = 784\,\text{J}$$

1.1.2 Types Of Energy

Broadly speaking, all energies can be classified into one of two types: **kinetic** or **potential.**

Kinetic Energy K is associated with motion. For an object of mass m moving at speed v, it is defined as:

$$K = \frac{1}{2}mv^2 \tag{3}$$

Gravitational Potential Energy U_g is associated with objects at a certain height above a reference point. For an object with mass m at height h above/below a reference point, it is defined as

$$U_\text{g} = mgh \tag{4}$$

Elastic Potential Energy U_e is associated with an extended/compressed spring. For a spring with spring constant k at an extension/compression x, it is defined as

$$U_\text{e} = \frac{1}{2}kx^2 \tag{5}$$

1.1.3 Conservation Of Energy

When a system is **isolated** (meaning that it does not exchange energy with other systems or its surroundings), then the **total energy is conserved.** However, the system is free to transform energies from one type to another (as long as the total energy is conserved).

In general, if total energy is not conserved, we need to account for the **work done by external forces** in the conservation of energy equation, which we call COE.

Example 1.2. A ball of mass m starts at the base of a rough incline at angle θ and coefficient of friction μ. If it is thrown up the incline with an initial speed of v parallel to the incline, what is the maximum distance it travels along the incline before stopping?

Solution. The initial kinetic energy of the ball is converted to gravitational potential energy as it gains height. However, since there is kinetic friction, some of the energy is also "wasted" due to the work done by friction, which we need to account for in our COE equation:

$$\frac{1}{2}mv^2 + W_\text{fric} = mgh$$

Importantly, W_{fric} is negative, because the kinetic friction force points down the incline, which is opposite to how the ball moves up the incline.

Let d be the distance moved up the incline. From Equation (1), we can find W_{fric}:

$$W_{\text{fric}} = -fd = -\mu N d = -\mu mgd\cos\theta$$

since $N = mg\cos\theta$, by balancing forces perpendicular to the incline.

Also, $h = d\sin\theta$. Hence, the COE equation reads:

$$\frac{1}{2}mv^2 - \mu mgd\cos\theta = mgd\sin\theta \qquad \Longrightarrow \qquad d = \frac{v^2}{2g\left(\sin\theta + \mu\cos\theta\right)}$$

Example 1.3. A spring with spring constant $k = 2000\,\text{N/m}$ has an unstretched length of 1.0 m. A man stretches it from 1.1 m to 1.2 m. Find the work done by the man.

Solution. The work done by the man is the change in the elastic potential energy of the spring:

$$W = \Delta U_{\text{e}} = \frac{1}{2}k\left({x_{\text{f}}}^2 - {x_{\text{i}}}^2\right) = \frac{1}{2}(2000)\left((1.2-1.0)^2 - (1.1-1.0)^2\right) = 30\,\text{J}$$

1.1.4 Work-Energy Theorem

Doing work is a form of transferring energy. **Positive work done** can be viewed as transferring an amount of energy into a system. **Negative work done** can be viewed as transferring an amount of energy out of a system.

They are related through the **work-energy theorem:**

$$W_{\text{net}} = \Delta K = \frac{1}{2}m\left({v_{\text{f}}}^2 - {v_{\text{i}}}^2\right) \tag{6}$$

where W_{net} is the net work done by **all forces** on the object.

Example 1.4. An eastward 20 N force acts for 10 m on a 1 kg mass, which is initially moving at 10 m/s. (a) If the mass initially moves eastward, find the final speed of the mass. (b) What if the mass initially moves westward?

Solution. (a) By a simple application of Equations (1) and (6), we have:

$$W_{\text{net}} = Fd = (20)(10) = 200\,\text{J}$$

$$200 = \frac{1}{2}(1)\left({v_{\text{f}}}^2 - 10^2\right) \qquad \Longrightarrow \qquad v_{\text{f}} = \sqrt{500}\,\text{m/s} = 22.4\,\text{m/s}$$

(b) This case is not as simple. We first need to check whether the mass stops at some point, since it might happen because the force is acting opposite in direction to the speed of the initial direction where the mass is moving.

Indeed, it happens, because the initial kinetic energy of the mass is

$$K_{\text{i}} = \frac{1}{2}m{v_{\text{i}}}^2 = \frac{1}{2}(1)(10)^2 = 50\,\text{J}$$

which is less than the net work done by the force.

When the mass first stops, there is $200 - 50 = 150\,\text{J}$ of work "remaining" by the force, which is then used to make the mass start moving eastward. Hence, we can now apply Equation (6) with the "remaining" work:

$$150 = \frac{1}{2}(1)\,{v_{\text{f}}}^2 \qquad \Longrightarrow \qquad v_{\text{f}} = \sqrt{300}\,\text{m/s} = 17.3\,\text{m/s}$$

1.1.5 Power

Power is defined as the amount of energy or work transferred or converted per unit time:

$$P = \frac{E}{t} \quad \text{or} \quad P = \frac{W}{t} \tag{7}$$

Just like the kinematics quantities, we should also be careful in distinguishing between instantaneous and average power.

If the force is **constant,** then the average power is:

$$P_{\text{ave}} = \frac{Fd}{t} = F\frac{d}{t} = Fv_{\text{ave}} \tag{8}$$

If the force is **not constant,** then we can only look at the instantaneous power:

$$P_{\text{inst}} = F_{\text{inst}}v_{\text{inst}} \tag{9}$$

Example 1.5. A vehicle weighs 10000 kg and accelerates from rest by an engine of constant power. After 40 s, it travels 400 m. Assume that the coefficient of kinetic friction is 0.05. The maximum speed is reached at 40 s. What is the maximum speed of the car? Take $g = 10\,\text{m/s}^2$.

Solution. When maximum speed is reached, the force exerted by the engine has matched the constant frictional force on the vehicle. This way, the vehicle is in equilibrium. Hence, we have:

$$F_{\text{engine}} = f \quad \Longrightarrow \quad \frac{P}{v} = \mu mg \quad \Longrightarrow \quad P = \mu mgv$$

At the same time, COE gives us (accounting for work done by friction and work done by the engine):

$$Pt + W_{\text{fric}} = W_{\text{engine}} \quad \Longrightarrow \quad Pt - \mu mgd = \frac{1}{2}mv^2 \quad \Longrightarrow \quad \mu mgvt - \mu mgd = \frac{1}{2}mv^2$$

This is a quadratic equation, and you can solve for v by plugging in the relevant variables. You will get $v = 20\,\text{m/s}$ as the only physically sound solution.

1.2 Impulse, Momentum & Collisions

Now, let's apply the dynamics we have learnt to analyse collisions. Before we do that, we need to introduce some more quantities.

1.2.1 Momentum & Impulse

The **momentum** $\boldsymbol{p}$ of a mass m moving at a velocity $\boldsymbol{v}$ is given by:

$$\boldsymbol{p} = m\boldsymbol{v} \tag{10}$$

Now that we know what momentum is, we can actually write a more accurate version of N2L. The more correct version is

$$\boldsymbol{F}_{\text{net}} = \frac{\Delta \boldsymbol{p}}{\Delta t} \tag{11}$$

If the **mass is changing,** you will need to use Equation (11) instead of $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$. A change in mass will also contribute to a change in momentum, which is not captured by $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$! Notice that Equation (11) reduces to $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$ when mass is constant.

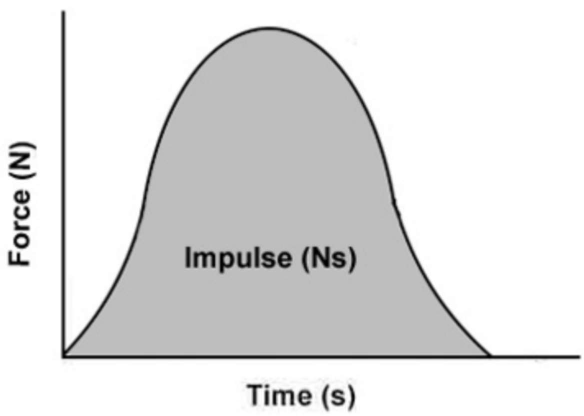
The **impulse $\boldsymbol{J}$** of a constant force $\boldsymbol{F}$ acting over a time interval t is given by:

$$\boldsymbol{J} = \boldsymbol{F}t \tag{12}$$

Like work, if the force $\boldsymbol{F}$ is not constant, but is rather constant over intervals, we can write:

$$\boldsymbol{J} = \sum_i \boldsymbol{F}_i t_i \tag{13}$$

Graphically, $|\boldsymbol{J}|$ is the area under the force-time graph:



1.2.2 Impulse-Momentum Theorem

Impulse and momentum are directly related through the **impulse-momentum theorem:**

$$\boldsymbol{J} = \Delta \boldsymbol{p} \tag{14}$$

This sort of reminds us of the work-energy theorem from Equation (6). Hence, think of impulse-momentum when a force acts over time and think of work-energy when a force acts over distance!

Example 1.6. Redo Example 1.1, but this time, using the idea of impulse and momentum.

Solution. Since we are given the forces and the times, we can directly find the total impulse as per Equation (13):

$$J = F_{0\text{ to }2}t_{0\text{ to }2} + F_{2\text{ to }5}t_{2\text{ to }5} + F_{5\text{ to }10}t_{5\text{ to }10} = (2)(2) + (4)(3) + (8)(5) = 56\,\text{Ns}$$

By the impulse-momentum theorem, this is the same as the change in momentum. Since the object was initially at rest, we have:

$$J = \Delta p = m\left(v_\text{f} - 0\right) = mv_\text{f} \quad\implies\quad v_\text{f} = \frac{J}{m} = \frac{56}{2} = 28\,\text{m/s}$$

By the work-energy theorem, the work done is equal to the change in kinetic energy, hence:

$$W = \Delta K = \frac{1}{2}m\left({v_\text{f}}^2 - 0\right) = \frac{1}{2}m{v_\text{f}}^2 = \frac{1}{2}(2)(28)^2 = 784\,\text{J}$$

which matches what we got in Example 1.1. Observe that the impulse-momentum approach was much faster, since we skipped all the pesky kinematics in between!

1.2.3 Conservation Of Momentum

When a system has **no net external force** acting on it, then the **total momentum is conserved.** However, the system is free to transfer momentum from one body to another (within the system).

This will be a fundamental principle when we deal with collisions, which we shall sometimes refer to as COM.

1.2.4 Types Of Collisions

We will deal primarily with **two-body collisions.** Before we go into the equations, let's take a look at the different types of collisions:

1. **Elastic Collision:** This is an ideal situation whereby both the **kinetic energy** and the **momentum** of the system is conserved.
2. **Inelastic Collision:** This is a less ideal situation whereby some kinetic energy is lost, but the **momentum** of the system is conserved.
3. **Completely Inelastic Collision:** This is a special situation whereby the two masses **stick together** after collision, and the **momentum** of the system is conserved.
4. **Disintegration:** This is a special situation whereby a mass breaks into smaller pieces, causing a gain in kinetic energy, but the **momentum** of the system is conserved.

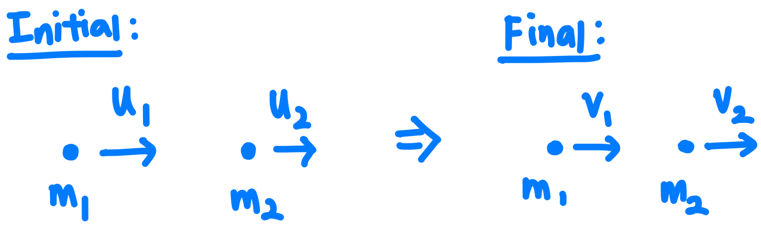
Observe that **regardless of the type of collision, the momentum of the system is conserved!**

1.2.5 Coefficient Of Restitution

Let's now look at a quantity that is important for characterising collisions: the **coefficient of restitution** C_{R} (also called COR). It is defined by:

$$C_{\mathrm{R}} = \frac{\text{relative speed of separation}}{\text{relative speed of approach}} = \frac{v_2 - v_1}{u_1 - u_2} \tag{15}$$

where u_1 and u_2 are the initial velocities of the first and second object, and v_1 and v_2 are the final velocities of the first and second object.



There are some important properties of the COR that you need to know:

1. The COR lies between 0 and 1, i.e. $0 \leq C_{\mathrm{R}} \leq 1$.
2. The COR is only defined for **collisions along one direction** (i.e. in 1D).
3. The COR depends only on the material properties of the two colliding objects.

It is instantly obvious that for **completely inelastic collisions,** $C_{\mathrm{R}} = 0$, since $v_1 = v_2$.

Example 1.7. Prove that for **elastic collisions,** $C_{\mathrm{R}} = 1$.

Solution. Consider the same two masses m_1 and m_2 colliding elastically, with velocities as defined in the diagram above. The two conservation laws we can apply for elastic collisions are COM and COE. Writing each equation, we have:

$$\text{COM:} \quad m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \Longrightarrow \quad m_1(u_1 - v_1) = m_2(v_2 - u_2)$$

$$\text{COE:} \quad \frac{1}{2}m_1{u_1}^2 + \frac{1}{2}m_2{u_2}^2 = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 \quad \Longrightarrow \quad m_1\left({u_1}^2 - {v_1}^2\right) = m_2\left({v_2}^2 - {u_2}^2\right)$$

At this point, since we want to get rid of the mass terms, we can take the rearranged form of COE and divide it by the rearranged form of COM. This also allows us to invoke the difference of squares formula, and hence we get:

$$\frac{m_1\left({u_1}^2 - {v_1}^2\right)}{m_1(u_1 - v_1)} = \frac{m_2\left({v_2}^2 - {u_2}^2\right)}{m_2(v_2 - u_2)} \quad \Longrightarrow \quad u_1 + v_1 = v_2 + u_2 \quad \Longrightarrow \quad C_\mathrm{R} = \frac{v_2 - v_1}{u_1 - u_2} = 1$$

as desired.

Since we derived the fact that $C_\mathrm{R} = 1$ for an elastic collision using COM + COE, this means that we are free to replace either one of COM or COE with the statement that $C_\mathrm{R} = 1$ **only for elastic collisions!** Usually, we will replace COE, since COE is quadratic in velocities, and we don't want to deal with quadratic equations.

Example 1.8. A ball is dropped from a height of H and collides with a hard, concrete ground. It rebounds to a height of $h < H$. Ignoring air resistance, find the coefficient of restitution associated with the collision with the ground.

Solution. We go back to the definition as per Equation (15). Here, the two objects are the ball and the ground. However, since the ground is hard (and much more massive than the ball), it is reasonable to assume that the ground stays stationary. Hence, the relative speeds of separation and approach are just the speeds of the ball!

By using COE or kinematics, we hence have:

$$C_\mathrm{R} = \frac{v_\text{ball, after impact}}{v_\text{ball, before impact}} = \frac{\sqrt{2gh}}{\sqrt{2gH}} = \sqrt{\frac{h}{H}}$$

1.2.6 2D Collisions

When it comes to 2D collisions, the same principles apply. However, since momentum is a vector quantity, we need to consider COM in two directions (usually the x and y-directions)!

Example 1.9. Two skaters collide and embrace. Skater A is 83 kg and was moving east at 6.2 m/s, while Skater B is 55 kg and was moving north at 7.8 m/s. (a) Find the velocity of the two skaters after impact. (b) What fraction of the initial kinetic energy was lost due to the collision?

Solution. (a) Set up a coordinate system such that the positive x-direction is eastward and the positive y-direction is northward. Then, in terms of vectors, we can write COM:

$$\boldsymbol{p}_\mathrm{A} + \boldsymbol{p}_\mathrm{B} = \boldsymbol{p} \quad \Longrightarrow \quad m_\mathrm{A}\boldsymbol{v}_\mathrm{A} + m_\mathrm{B}\boldsymbol{v}_\mathrm{B} = (m_\mathrm{A} + m_\mathrm{B})\,\boldsymbol{v}$$

$$\Longrightarrow \quad 83\begin{pmatrix}6.2\\0\end{pmatrix} + 55\begin{pmatrix}0\\7.8\end{pmatrix} = (83 + 55)\begin{pmatrix}v_x\\v_y\end{pmatrix} \quad \Longrightarrow \quad \begin{pmatrix}v_x\\v_y\end{pmatrix} = \begin{pmatrix}3.729\\3.109\end{pmatrix}$$

Hence, $|\boldsymbol{v}| = \sqrt{3.729^2 + 3.109^2} = 4.86\,\text{m/s}$, and $\theta = \tan^{-1}\left(\frac{3.109}{3.729}\right) = 39.8^\circ$ north of east.

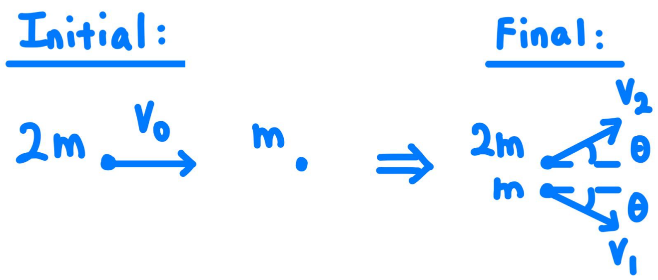
(b) Using the final speed, we have:

$$\text{Fraction Of Initial KE Lost} = \frac{\frac{1}{2}m_{\mathrm{A}}{v_{\mathrm{A}}}^2 + \frac{1}{2}m_{\mathrm{B}}{v_{\mathrm{B}}}^2 - \frac{1}{2}(m_{\mathrm{A}}+m_{\mathrm{B}})\,v^2}{\frac{1}{2}m_{\mathrm{A}}{v_{\mathrm{A}}}^2 + \frac{1}{2}m_{\mathrm{B}}{v_{\mathrm{B}}}^2} = 1 - \frac{(m_{\mathrm{A}}+m_{\mathrm{B}})\,v^2}{m_{\mathrm{A}}{v_{\mathrm{A}}}^2 + m_{\mathrm{B}}{v_{\mathrm{B}}}^2}$$

$$= 1 - \frac{(83+55)\,(4.86)^2}{(83)\,(6.2)^2 + (55)\,(7.8)^2} = 0.502$$

Example 1.10. A mass $2m$ moving at initial speed v_0 collides elastically with a stationary mass m. If the two masses scatter at equal angles with respect to the incident direction, what is this angle?

Solution. We first draw a diagram to illustrate the collision:



Since this is an elastic collision, we can write COM (in both the x and y-directions) and COE. Our equations are:

$$\text{COM in } x\text{-direction:} \quad 2mv_0 = mv_1\cos\theta + 2mv_2\cos\theta$$

$$\text{COM in } y\text{-direction:} \quad mv_1\sin\theta = 2mv_2\sin\theta$$

$$\text{COE:} \quad \frac{1}{2}(2m)\,v_0^2 = \frac{1}{2}m{v_1}^2 + \frac{1}{2}(2m)\,{v_2}^2$$

At this juncture, we have 3 equations and 3 unknowns (v_1, v_2 and θ), which means we are ready to solve. After some work (left as an exercise), you will get $\theta = 30°$.

1.3 Centre Of Mass

We know that the total momentum of a system of particles is equal to the vector sum of each of their individual momenta. However, we can also think of the total momentum as such:

$$\sum_i \boldsymbol{p}_i = \left(\sum_i m_i\right)\boldsymbol{v}_{\mathrm{CM}} = M\boldsymbol{v}_{\mathrm{CM}} \tag{16}$$

This is equivalent to taking the momentum of an imaginary particle with a mass equal to the total mass M of all the particles, moving at some velocity $\boldsymbol{v}_{\mathrm{CM}}$.

This imagination is the basis of the concept of the **centre of mass,** or CM for short.

1.3.1 Calculating Centre Of Mass

The **position** of the centre of mass can be found by considering a "weighted average" of each particle's position:

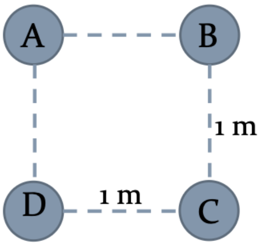
$$\boldsymbol{r}_{\mathrm{CM}} = \frac{\sum_i m_i\boldsymbol{r}_i}{\sum_i m_i} \tag{17}$$

The position of the centre of mass is where the effective total gravitational force acts on the system of particles, in a uniform gravitational field.

The **velocity** of the centre of mass is, as per Equation (16), given by:

$$\boldsymbol{v}_{\mathrm{CM}} = \frac{\sum_i \boldsymbol{p}_i}{\sum_i m_i} = \frac{\sum_i m_i \boldsymbol{v}_i}{\sum_i m_i} \tag{18}$$

Example 1.11. The masses of A, B, C and D in the figure below are 1 kg, 2 kg, 3 kg and 4 kg respectively. (a) Find the position of the centre of mass. (b) If the four particles are moving with velocities (in m/s) of $\begin{pmatrix}4\\2\end{pmatrix}$, $\begin{pmatrix}2\\-2\end{pmatrix}$, $\begin{pmatrix}0\\2\end{pmatrix}$ and $\begin{pmatrix}-2\\0\end{pmatrix}$ respectively, find the velocity of the centre of mass. Does the velocity of the centre of mass depend on the positions of the particles?



Solution. (a) Let's put an arbitrary origin $(0, 0)$ at D, so that we can measure lengths with respect to this origin. Let the x and y-directions be the horizontal and vertical directions respectively.

We can solve for $\boldsymbol{r}_{\mathrm{CM}}$ by directly applying Equation (17):

$$\boldsymbol{r}_{\mathrm{CM}} = \frac{\sum_i m_i \boldsymbol{r}_i}{\sum_i m_i} = \frac{m_{\mathrm{A}}\boldsymbol{r}_{\mathrm{A}} + m_{\mathrm{B}}\boldsymbol{r}_{\mathrm{B}} + m_{\mathrm{C}}\boldsymbol{r}_{\mathrm{C}} + m_{\mathrm{D}}\boldsymbol{r}_{\mathrm{D}}}{m_{\mathrm{A}} + m_{\mathrm{B}} + m_{\mathrm{C}} + m_{\mathrm{D}}}$$

$$= \frac{1}{1+2+3+4}\left(1\begin{pmatrix}0\\1\end{pmatrix} + 2\begin{pmatrix}1\\1\end{pmatrix} + 3\begin{pmatrix}1\\0\end{pmatrix} + 4\begin{pmatrix}0\\0\end{pmatrix}\right) = \begin{pmatrix}0.5\\0.3\end{pmatrix}\mathrm{m}$$

Hence, the centre of mass is located at 0.5 m right and 0.3 m top of D.

(b) We can solve for $\boldsymbol{v}_{\mathrm{CM}}$ by directly applying Equation (18):

$$\boldsymbol{v}_{\mathrm{CM}} = \frac{\sum_i m_i \boldsymbol{v}_i}{\sum_i m_i} = \frac{1}{1+2+3+4}\left(1\begin{pmatrix}4\\2\end{pmatrix} + 2\begin{pmatrix}2\\-2\end{pmatrix} + 3\begin{pmatrix}0\\2\end{pmatrix} + 4\begin{pmatrix}-2\\0\end{pmatrix}\right) = \begin{pmatrix}0\\0.4\end{pmatrix}\mathrm{m/s}$$

We can see that $\boldsymbol{v}_{\mathrm{CM}}$ doesn't depend on the positions of the particles, since we didn't use them for part (b) at all!

1.3.2 Using Centre Of Mass

The CM can be useful in helping us solve problems, because the CM obeys Newton's Laws.

If **no net external force** acts on a system of particles, and **the CM is initially at rest,** then t**he CM must always remain at rest,** i.e. the position of the CM is conserved! This is sometimes termed the conservation of CM.

Example 1.12. John (of mass 60 kg) and James (of mass 90 kg) are standing 20 m apart on an ice skating rink (frictionless ground). They pull on each other with a light rope. When James has moved 6 m towards their midpoint, how much did John move?

Solution. The tension T acting on John and James are internal forces that cancel out, if we treat John + James as our system. Because they were both initially stationary, the position of the CM of John and James will remain in the same position, no matter how each of them moves!

Let us take the initial position of John as the reference point, and let James be 20 m to the right at first. Then, the position of the CM is:

$$x_{\text{CM}} = \frac{m_{\text{John}} x_{\text{John}} + m_{\text{James}} x_{\text{James}}}{m_{\text{John}} + m_{\text{James}}} = \frac{(60)(0) + (90)(20)}{60 + 90} = 12\,\text{m, right of John}$$

When James moved by 6 m, $x'_{\text{James}} = 20 - 6 = 14\,\text{m}$. Suppose John is now at a position x'_{John} with respect to his initial position. Using the conservation of CM,

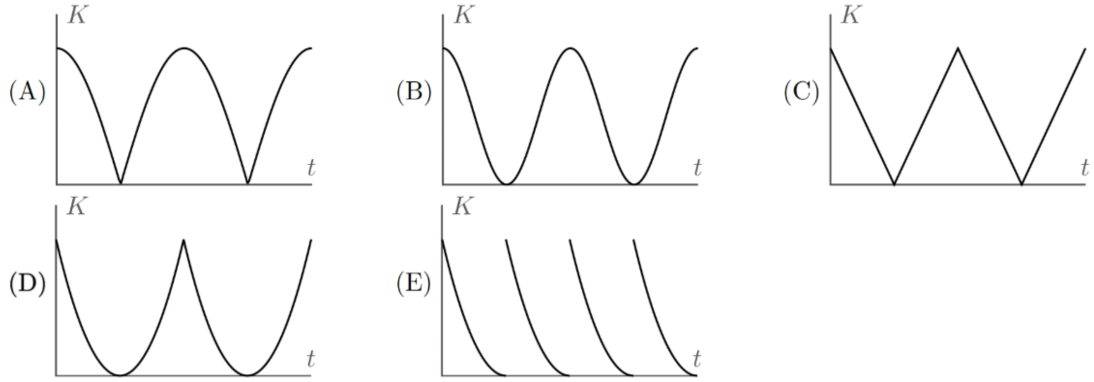
$$12 = \frac{m_{\text{John}} x'_{\text{John}} + m_{\text{James}} x'_{\text{James}}}{m_{\text{John}} + m_{\text{James}}} = \frac{60 x'_{\text{John}} + (90)(14)}{60 + 90} \quad \Longrightarrow \quad x'_{\text{John}} = 9\,\text{m}$$

Hence, John has moved by 9 m towards their midpoint, to keep their CM at the same position.

Importantly, using this method, we didn't have to care about *how* John and James pulled each other! The answer will always be the same.

2 Problems

Problem 2.1 ($F = ma$ 2024)**.** A bouncy ball is thrown vertically upward from the ground. Air resistance is negligible, and the ball's collisions with the ground are perfectly elastic. Which of the following shows the kinetic energy of the ball as a function of time? Assume the collisions are too quick for their duration to be seen in the plot.



Solution. (D)

Problem 2.2 ($F = ma$ 2020)**.** A conveyor belt is moving with velocity v to the east. A block with velocity v to the south slides from the ground onto the conveyor belt. The coefficient of friction between the block and the belt is μ. At what time later (taking $t = 0$ to be the time when the block first slides onto the conveyor belt) does the block stop slipping?

(A) $\frac{v}{\sqrt{2}\mu g}$

(B) $\frac{v}{\mu g}$

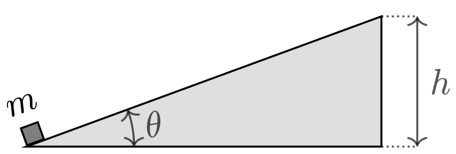
(C) $\frac{\sqrt{2}v}{\mu g}$

(D) $\frac{2v}{\mu g}$

(E) The block never stops slipping.

Solution. (C)

Problem 2.3 ($F = ma$ 2023)**.** A box of mass m is at the bottom of an inclined plane with angle θ to the horizontal, and height h.



A person drags the box very slowly up the plane, by applying a force parallel to the plane. The coefficient of kinetic friction between the box and the plane is μ_k. When the box reaches the top of the plane, how much work has the person done?

(A) $mgh\,(1 + \mu_k \sin\theta)$

(B) $mgh\,(1 + \mu_k \cos\theta)$

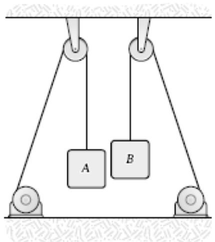
(C) $mgh\,(1 + \mu_k \tan\theta)$

(D) $mgh\,(1 + \mu_\mathrm{k} \csc\theta)$

(E) $mgh\,(1 + \mu_\mathrm{k} \cot\theta)$

Solution. (E)

Problem 2.4 (SJPO 2009)**.** Two equal masses are raised at constant speeds by a pulley system as shown. Mass B is raised twice as fast as mass A. The magnitudes of the forces are F_A and F_B, while the power supplied is respectively P_A and P_B. Which of the following is true?



(A) $F_\mathrm{B} = F_\mathrm{A}$, $P_\mathrm{B} = P_\mathrm{A}$

(B) $F_\mathrm{B} = F_\mathrm{A}$, $P_\mathrm{B} = 2P_\mathrm{A}$

(C) $F_\mathrm{B} = 2F_\mathrm{A}$, $P_\mathrm{B} = P_\mathrm{A}$

(D) $F_\mathrm{B} = 2F_\mathrm{A}$, $P_\mathrm{B} = 2P_\mathrm{A}$

(E) $F_\mathrm{B} = 2F_\mathrm{A}$, $P_\mathrm{B} = 4P_\mathrm{A}$

Solution. (B)

Problem 2.5 (SJPO 2011)**.** A tennis ball bounces on the floor three times. If each time it loses 23.0% of its energy due to heating, how high does it bounce after the third time, given that we released it 4.90 m from the floor? Ignore other energy losses such as air resistance.

(A) 0.0596 m

(B) 0.259 m

(C) 1.13 m

(D) 2.24 m

(E) 2.91 m

Solution. (D)

Problem 2.6 (SJPO 2010)**.** A block of mass m is hung from a light vertical spring of spring constant k, which is hung in turn from another identical spring. The amount by which each spring stretches is x. What is the total elastic potential energy of the system when at rest?

(A) $\frac{1}{2}mgx$

(B) mgx

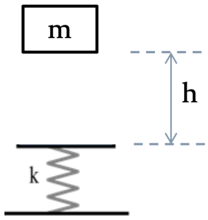
(C) $2mgx$

(D) $\frac{m^2g^2}{2k}$

(E) $\frac{2m^2g^2}{k}$

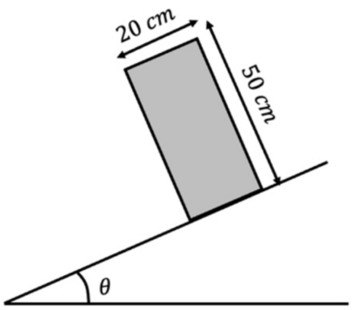
Solution. (B)

Problem 2.7. A 1 kg mass falls from a height of 2 m onto a light platform supported by a spring of $k = 500\,\text{N/m}$. (a) What is the maximum elastic potential energy in the spring? (b) What is the maximum speed of the mass?



Solution. a) $22.6J$ b) $6.26ms^{-1}$

Problem 2.8 (SJPO 2024, unused)**.** A uniform rectangular block rests on a very rough inclined plane. What is the maximum angle of tilt θ before the block tilts over?



(A) 11.3°

(B) 68.2°

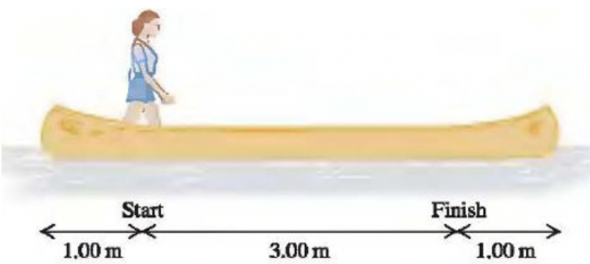
(C) 51.3°

(D) 21.8°

(E) 38.7°

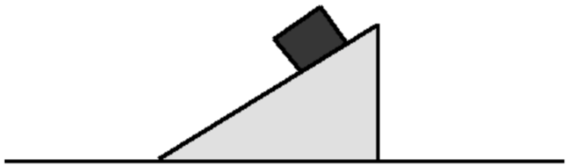
Solution. (D)

Problem 2.9. A 45 kg woman stands up in a 60 kg canoe 5 m long. She walks from a point 1 m from one end to a point 1 m from the other end. If you ignore resistance to motion of the canoe in the water, how far does the canoe move in the process?



Solution. 1.57m

Problem 2.10 ($F = ma$ 2007). A large wedge rests on a horizontal frictionless surface, as shown. A block starts from rest and slides down the inclined surface of the wedge. During the motion of the block, how does the centre of mass of the block and wedge move?



(A) It does not move.

(B) It moves horizontally with constant speed.

(C) It moves horizontally with increasing speed.

(D) It moves vertically with increasing speed.

(E) It moves both horizontally and vertically.

Solution. (A)

Problem 2.11 (SJPO 2008). A large truck and a small car collided near Bukit Timah road one day and the two vehicles were stuck together. Which vehicle has undergone a larger change in momentum?

(A) The car.

(B) The truck.

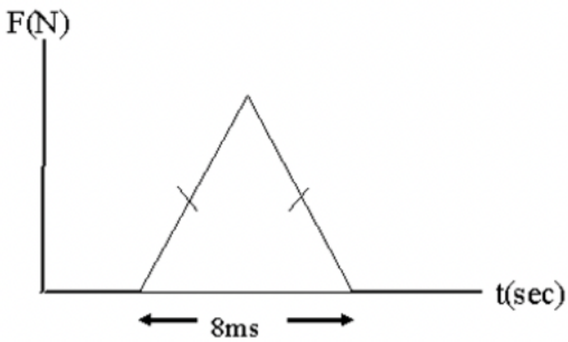
(C) The momentum change was the same for both vehicles.

(D) We cannot tell without knowing the final velocity of the combined mass.

(E) We cannot tell without knowing the masses of the truck and the car.

Solution. (C)

Problem 2.12 (SJPO 2010). A ball of mass 0.25 kg is thrown with speed of 30 m/s. The ball strikes a bat and it is hit straight back along the same line at a speed 50 m/s. The variation of the interaction force with the ball is shown as an isosceles triangle below. What is the maximum force exerted by the bat on the ball?



(A) 2500 N

(B) 5000 N

(C) 7500 N

(D) 1250 N

(E) 1000 N

Solution. (B)

Problem 2.13 (SJPO 2011). An 8.0 g bullet is shot into a 4.0 kg block, at rest on a frictionless horizontal surface. The bullet remains lodged in the block. The block moves towards a spring and compresses it by 9.4 cm. The spring constant of the spring is 1000 N/m. Which of the following is the initial speed of the bullet closest to?

(A) 710 m/s

(B) 740 m/s

(C) 770 m/s

(D) 800 m/s

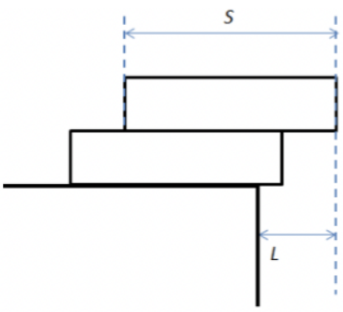
(E) 830 m/s

Solution. (B)

Problem 2.14. A platform scale is calibrated to indicate the mass in kg of an object placed on it. Particles fall from a height of 3.5 m and collide with the balance pan of the scale. Assuming that the collisions are elastic and the particles rebound upwards with the same speed they had before hitting the pan, determine the average scale reading if each particle has mass 110 g and collisions occur at a rate of $42\,\mathrm{s}^{-1}$.

Solution. 7.81kg

Problem 2.15 (SJPO 2012). Two identical rectangular bricks of length s are piled with one on top of the other on a table. What is the maximum distance L (see the figure below) that the top brick can be extended beyond the edge of the table for the system to remain balanced?



(A) $\frac{1}{2}S$

(B) $\frac{2}{3}S$

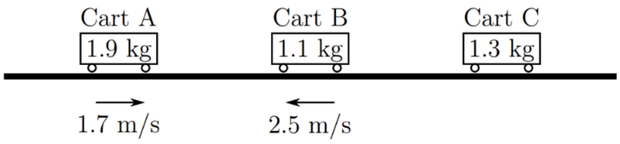
(C) $\frac{3}{4}S$

(D) $\frac{7}{8}S$

(E) S

Solution. (C) Hint: what is the maximum distance that the top brick can extend over the bottom one

Problem 2.16 ($F = ma$ 2015). Three trolley carts are free to move on a 1D frictionless horizontal track. Cart A has a mass of 1.9 kg and an initial speed of 1.7 m/s to the right. Cart B has a mass of 1.1 kg and an initial speed of 2.5 m/s to the left. Cart C has a mass of 1.3 kg and is originally at rest. Collisions between carts A and B are perfectly elastic, while collisions between carts B and C are perfectly inelastic. What is the velocity of the centre of mass of the system of the three carts after the last collision?



(A) 0.11 m/s

(B) 0.16 m/s

(C) 1.4 m/s

(D) 2.0 m/s

(E) 3.23 m/s

Solution. (A) Hint: is there an external force on the system?

Problem 2.17 ($F = ma$ 2015). A 0.650 kg ball moving at 5.00 m/s collides with a 0.750 kg ball that is originally at rest. After the collision, the 0.750 kg moves off with a speed of 4.00 m/s, and the 0.650 kg ball moves off at a right angle to the final direction of motion of the 0.750 kg ball. What is the final speed of the 0.650 kg ball?

(A) 1.92 m/s

(B) 2.32 m/s

(C) 3.00 m/s

(D) 4.64 m/s

(E) 5.77 m/s

Solution. (A) Hint: Pythagoras' theorem

Problem 2.18 ($F = ma$ 2013). Jordi stands 20 m from a wall and Diego stands 10 m from the same wall, on the same side. Jordi throws a ball at an angle of 30° above the horizontal, and it collides elastically with the wall. How fast does Jordi need to throw the ball so that Diego will catch it? Consider Jordi and Diego to be the same height, and both are on the same perpendicular line from the wall. *There is a very elegant way to do this problem.*

(A) 11 m/s

(B) 15 m/s

(C) 19 m/s

(D) 30 m/s

(E) 35 m/s

Solution. (C) Hint: Think in terms of 2 paths from the wall to Jordi and Diego

Problem 2.19 (SJPO 2013). Automobiles are made to be energy-absorbing in the event of a crash. It means that the components of the body are designed to be anything but elastic, and to be sacrificed while dissipating kinetic energy. Sometimes this is described loosely by the size of the crumple-zone ahead or behind the passengers of the car. A Datsun had a crumple-zone of roughly 1.5 m, and about 1 m of it had been used up when it hit a concrete wall at about 30 km/h. What is the minimum force needed to bring the 1000 kg car to rest in this distance?

(A) 2.3×10^4 N

(B) 4.5×10^6 N

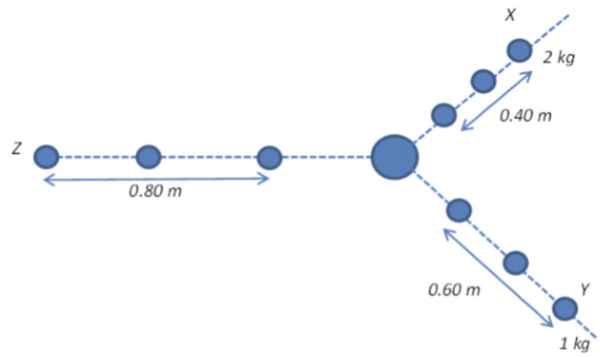
(C) 3.5×10^4 N

(D) 450 N

(E) 30 N

Solution. (C)

Problem 2.20 (SJPO 2014). A body at rest explodes breaking into three pieces which move off at different velocities, all in the same horizontal plane. The following figure shows the experimental results as drawn from a stroboscopic photograph of the event. The time interval between flashes was 0.10 s. The pieces X and Y travel at right angles to each other, and their masses are 2.0 kg and 1.0 kg respectively. Piece Z was unfortunately lost after the explosion. What is the mass of Piece Z?



(A) 1.25 kg

(B) 1.50 kg

(C) 2.00 kg

(D) 0.85 kg

(E) 3.00 kg

Solution. (A)